Physics in Medicine:
Fundamentals of Analyzing Biomedical Signals

Klaus Lehnertz

Topics:
- theory of nonlinear dynamical systems
- characterizing measures
- biosignals and recording of biosignals
- applications (medicine, physics, biology)

http://epileptologie-bonn.de/cms/front_content.php?idcat=203
Literature:

Physics:

Medicine:
Literature:

**Signal processing, Statistics, Computing:**

TISEAN
Nonlinear Time Series Analysis

Rainer Hegger
Holger Kantz
Thomas Schreiber

Go to Version 3.0.1 (released March 2007)

Go to Version 2.1 (released December 2000)
Fundamentals of Analyzing Biomedical Signals

Introduction

Historical overview:

1778  P. Laplace (Laplace’s demon, “everything is predictable”)
1880  W. Sierpinski (non-Euclidian geometry, “mathematical monster”)
1892  H. Poincare (three-body problem, dimension of manifolds)
1919  F. Hausdorff (extension of notion of dimension)
1963  E. Lorenz (weather forecasting)
1967  B. Mandelbrot (fractals, self similarity)
1975  J. Yorke (deterministic chaos)
1977/78 routes into chaos:
   S. Grossmann/S. Thomae (period doubling)
   M. Feigenbaum (Feigenbaum constant),
   Newhouse-Ruelle-Takens route
1981  D. Ruelle (strange attractors),
   P. Grassberger / I. Procaccia (correlation dimension)
   F. Takens (state space reconstruction)

since 1990 nonlinear time series analysis
Time series analysis

Goals

- knowledge about some system
- expand
- specify
- future (prediction)
- past (analysis)
**Fundamentals of Analyzing Biomedical Signals**

**Introduction**

**Time series analysis**

- system
- measurement device
- observable (time series)
- environment

**note: interactions !**

- what is a suitable device ?
- what is a good observable ?
- what is a suitable environment ?
- interfaces ?
**Fundamentals of Analyzing Biomedical Signals**

**Introduction**

**Time series analysis**

* time series: - sequence of data (length $N$)  
  - measurement or simulation (model)  
  - time-dependent

\[
\left( v_i, v_{i+\Delta t}, \ldots, v_{N\Delta t} \right)
\]

$\Delta t$ - temporal distance between successive data points  
- sampling interval (measurement)
<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>Model Simulation</th>
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</thead>
<tbody>
<tr>
<td><strong>Length of Time Series</strong></td>
<td>(mostly) limited</td>
<td>user-defined</td>
</tr>
<tr>
<td><strong>Sampling Interval</strong></td>
<td>limited</td>
<td>user-defined</td>
</tr>
<tr>
<td><strong>Precision</strong></td>
<td>A/D converter noise</td>
<td>user-defined</td>
</tr>
</tbody>
</table>
Fundamentals of Analyzing Biomedical Signals

Introduction

System

interactions: system – environment?

open system?

isolated system?
Dynamical system

- system under influence of some force ($\delta\nu\nu\alpha\mu\imath\omega = \text{force}$)
- **time-dependent** system states
- state changes depend on current state

**Deterministic**
- same initial states
  - same evolution

**Stochastic**
- same initial states
  - random evolution
Dynamical system

- characterized by time-dependent state variables $\mathbf{x}(t) \in \mathbb{R}^d$
- temporal evolution of state variables:

*continuous case*: set of (first-order) ordinary differential equations with initial conditions $\mathbf{x}(0) = \mathbf{x}(t_0)$

$$\frac{d\mathbf{x}(t)}{dt} = f(t, \mathbf{x}(t), \beta)$$

*discrete case*: set of difference equations (mapping) with initial conditions $\mathbf{x}_0 = \mathbf{x}_{t_0}$

$$\mathbf{x}_{t+\Delta t} = F(T, \mathbf{x}_t, \beta)$$

with $d = $ dimension of system; $\beta = $ control parameter; $f, F = $ nonlinear functions in case of nonlinear systems
Nonlinearity - Linearity

linear

simple

equations
can have

solutions

nonlinear

complex

impact of changing control parameters or initial conditions?

system

linear

proportional

nonlinear

non-proportional

effect
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Introduction

- Condensed matter physics
  - Pattern formation
  - Phase transitions
  - Spin waves

- Fluid mechanics
  - Transition to turbulent motion
  - Crystal growth
  - Surface of liquids

- Laser physics
  - Laser instabilities
  - Semiconductor laser
  - Coupled laser

- Mechanics
  - Nonlinear oscillators
  - Coupled/forced pendulums
  - Magneto-mechanical oscillators
  - Torsion bar

- Acoustics
  - Sound generation with:
    - Laser
    - Musical instruments

- Nonlinear dynamics in physics

- Astrophysics
  - Solar system
  - Motion of stars
  - Sun spots
  - Pulsar/quasar
  - Distribution of galaxies

- Plasma physics
  - Oscillations in gas discharges
  - Pattern formation
  - Plasma waves

- Optics
  - Opto-galvanic systems
  - Nonlinear optics
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Introduction

- biochemistry
- biological oscillations
- epidemiology
- ecology
- population dynamics
- genetics
- physiology
- nonlinear dynamics in medicine/biology
- neurology
- epileptology
- psychiatry
- psychology
- endocrinology
- pneumology
- cardiology
- nephrology
Fundamentals of Analyzing Biomedical Signals

Introduction

- **neuron:** highly nonlinear
- **neuron pools:** networks with complex interactions
  - electromagnetic, chemical, morphological
- **nonlinear (??), open, dissipative, adaptive**
- **approx. $10^{11}$ neurons**
  - each with $10^3 - 10^4$ synapses
Nonlinearity and Causality

physical phenomenon → reason? → strongly deterministic causality principle

(exactly) equal causes have (exactly) equal effects

past → now → future

time

math. model (ODE) +
initial condition

system predictable +
behavior reproducible
The pragmatic perspective of a linear nature

- system
- equations of motion
- model

Mathematical concepts:
- linear equations
- linear models
- remainder: irregularities, stochastic fluctuations

This perspective challenged by Poincaré and Sierpinski
Nonlinearity and Causality

weak causality:
\[ \text{equal causes } \rightarrow \text{equal effects} \]

strong causality:
\[ \text{similar causes } \rightarrow \text{similar effects} \]

strong idealization;
does not account for experimental conditions

includes weak causality;
accounts for experimental conditions:
tiny deviations from initial conditions,
weak perturbations, systematic errors, …
violation of strong causality:

*similar causes → vastly different effects*

- sensitive dependence on initial conditions
- deterministic chaos
- pattern formation
- “the whole is more than the sum of its parts” (Aristoteles)
- self-organization
# Processes and their Characteristics

<table>
<thead>
<tr>
<th>Regular Process</th>
<th>Chaotic Process</th>
<th>Stochastic Process</th>
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<tr>
<td>Deterministic</td>
<td>Deterministic</td>
<td>Stochastic (noise/randomness)</td>
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<tr>
<td>Long-term</td>
<td>Predictable</td>
<td>Non-predictable</td>
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<tr>
<td>Predictable</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strong causality</td>
<td>Violation of</td>
<td>No causal</td>
</tr>
<tr>
<td></td>
<td>strong causality</td>
<td>relationships</td>
</tr>
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<td></td>
<td>Nonlinearity</td>
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</tbody>
</table>
Deterministic Chaos

**Chaos** (colloquially)
- disordered state and irregularity

**Deterministic Chaos**
- irregular (non-periodic) behavior
- non-predictable or for some short time horizon only
- deterministic equations of motion (in contrast to stochasticity)
- instabilities and recurrences
Fundamentals of Analyzing Biomedical Signals

Introduction

Deterministic Chaos

dripping faucet

The Science Frontie Express Serie
The Dripping Faucet as a Model Chaotic System
by Robert Shaw
Deterministic Chaos

“chaotic behavior can arise from very simple non-linear dynamical equations”: **logistic map** (model for population growth, 1837)

\[ x_{n+1} = r x_n (1 - x_n); \quad x_n \in [0, 1]; \quad r \in [0, 4]\]
Deterministic Chaos

“chaotic behavior can arise from very simple non-linear dynamical equations”: **logistic map** (model for population growth, 1837)

\[ x_{n+1} = r x_n (1 - x_n); \quad x_n \in [0, 1]; \quad r \in [0, 4] \]

- period-doubling route to chaos
- period-3 implies chaos
- islands of stability
- periodicities within chaos
- self-similarity
Deterministic Chaos

Arnold’s cat map:

chaotic map from the torus into itself:

\[ \Gamma : \mathbb{T}^2 \rightarrow \mathbb{T}^2 \]

\[ \Gamma : (x, y) \rightarrow (2x + y, x + y) \mod 1 \]

deterministic operations:
- stretching
- bending
- folding (nonlinear)
Nonlinear dynamical systems

- can be described by nonlinear ODEs. However, no analytic solutions exist!

- show qualitatively rich dynamics:
  drastic changes upon changes of control parameters (bifurcation)
  deterministic chaos

- long-term behavior can be assessed by investigating the phase-space (state-space)