Compressed Sensing MRI
A look at how CS can improve on current imaging techniques

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Outline

- Introduction to Compressive Sensing
- CS in MRI
  - medical imagery is compressible in an appropriate transform domain
  - MRI scanners acquire encoded samples, rather than direct pixel samples
  1. Rapid 3-D angiography
  2. Whole-heart coronary imaging
  3. Brain imaging
  4. Dynamic heart imaging
Recovering signal \( f \in C^N \) from noisy linear measurement \( y = Af + n \in C^M \), \( A \in C^{M \times N} \) where \( M \ll N \) such that

1. the unknown signal \( f \) is \( K \)-sparse or compressible with \( K \) significant coefficient.
2. the noise process is bounded \( \|n\|_2 < \varepsilon \).

The CS theory guarantees stable solution if RIC \( \delta_K \) for \( A \) is the smallest \( \delta \in (0, 1) \) such that

\[
(1 - \delta_K) \|x\|_2^2 \leq \|Ax\|_2^2 \leq (1 + \delta_K) \|x\|_2^2
\]

holds all vector \( x \) with at most \( K \) non-zero entries.
Signal can be achieved through the $L_1$ inverse problem

$$\min_f \|f\|_1 \text{ subject to } \|Af - y\|_2 \leq \epsilon^2.$$ 

If $\delta_{2K} < \sqrt{2} - 1$ and $\|n\|_2 < \epsilon$ then the solution $\hat{f}$ to equation above satisfy

$$\|f^* - \hat{f}\|_2 \leq C_0 K^{-\frac{1}{2}} \|f^* - f_K\|_1 + C_1 \epsilon,$$

where $f_K$ is the best $K$-sparse approximation to the true solution $f^*$.

The optimization problem can be viewed as convex relaxation of NP of finding the sparsest solution

$$\min_f \|f\|_0 \text{ subject to } \|Af - y\|_2 \leq \epsilon^2.$$ 

where $L_0$ norm shows the number of non-zero entries in the vector.
For large $M$ estimating and testing the RIC is impractical.

A tractable bound on the RIC $(\delta_K \leq (K - 1)\mu(A))$ through the MUTUAL COHERENCE of the columns of $A$, $\mu(A) = \max_{i \neq j} |a_i^H a_j|$, can be used to guarantee a stable $L_1$ recovery ($M = \mathcal{O}(K \mu \log N)$).

Compressive Sensing:

1. linear model with low coherence among regressor.
2. low complexity nonlinear reconstruction algorithms.
3. sufficient conditions for stable reconstruction.
\[ s(t) = \int_{\mathbb{R}} m(\vec{r}) e^{-i2\pi \vec{k}(t) \cdot \vec{r}} \, dr \] where \( k(t) \propto \int_0^t G(s) \, ds \)
Sampling Techniques in MRI

Sampling scheme changes the image quality.
1. Transformation sparsity

2. Incoherence of undersampling artifact: the artifacts in reconstruction caused by undersampling should be incoherence (noise like) in the sparsity domain.

3. Nonlinear reconstruction

\[
\text{minimize } \| \psi m \|_1 \text{ s.t. } \| \mathcal{F}_S m - y \|_2 < \varepsilon
\]
Sparsity in MRI images

Mapping image content into a vector of sparse coefficients then consider the significant ones and ignore small coefficients.
Random sampling fulfills the incoherence condition.
How the random sampling cancels out the artifact and interference.
Point spread function is a measure for incoherence:

$$PSF(i, j) = (\mathcal{F}_s^* \mathcal{F}_s)(i, j)$$
We look at four potential applications of CS in MRI.

1. Rapid 3-D angiography
2. Whole-heart coronary imaging
3. Brain imaging
4. Dynamic heart imaging

The way in which different applications face different constraints, imposed by MRI scanning hardware or by patient considerations and how the inherent freedom of CS to choose sampling trajectories and sparsity transforms plays a role in matching the constraints.
Rapid 3-D Angiography

3-D Cartesian Sampling Configuration

Nyquist Sampling

Low Resolution

Linear

CS
Whole-Heart Coronary Imaging

[Diagram of whole-heart coronary imaging process with annotations and images showing different slices and imaging techniques.]
Multislice 2-D Cartesian Sampling Configuration

Nyquist Sampling

Low-Resolution Sampling

Linear

CS Wavelet + TV
Dynamic Heart Imaging

(a) Dynamic Imaging
(b) Traditional Dynamic $k$-Space Sampling
(c) Random Ordered $k$-Space Sampling

Simulation:
True
CS
Linear

Time (a)

Dynamic Cardiac Imaging:
Linear
CS

Cross Section
Time (b)

Time
Conclusions

1. Optimizing sampling trajectory.
2. Developing improved sparse transforms that are incoherence to sampling operator.
3. Studying reconstruction quality in terms of clinical significance.
4. Improving the speed of reconstruction algorithms.

In fact, implementation of this scheme requires only minor modifications to existing pulse sequences.