MRI: From Signal to Image

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Magnetic Resonance Tomography

- Tomography:
  - tomos: section
  - graphein: to write

- Signal measured as function of space
Signal

- Receiver coils placed with their normal direction perpendicular to $z$-axis
- Precessing transversal magnetic moment $m_T$ induces signal into receiver coils
- Flux $\Phi$ proportional to $m_T$
Magnetic moments precess with Larmor frequency $\omega_0 = \gamma B$

Received signal is caused by all precessing moments in the sample

Problem: No spatial differentiation of the signals

Solution

Spatial encoding using magnetic gradient fields
Slice Selection

- Apply gradient in $z$-direction during the HF pulse
- Larmor frequency: $\omega_0(z) = \gamma(B_0 + G_{ss}z)$
- Applying HF pulse with desired spectrum results in transversal magnetization in selected slice
- Usually slice with sharp transition is desired
- Envelope function of pulse approximates the sinc function
- Along the slice $m_T$ gets dephased
- Reverse gradient after excitation for half the excitation time → rephasing
Frequency Encoding

- Applying gradient (e.g. in $x$ direction) during readout
- Larmor frequency: $\omega_0(x) = \gamma(B_0 + G_{fe}x)$
- Amplitude of measured signal per frequency is projection of single slice
Magnetic Resonance Tomography
Spatial Encoding
Image Reconstruction

Slice Selection
Frequency Encoding
Phase Encoding

lower frequency + higher frequency = Total MR signal

Fourier transform

Signal amplitude vs. Frequency/position
- Works only in one spatial dimension
- Multiple gradients lead to ambiguous results
- Another mechanism must be used to encode second dimension: *Phase encoding*
Phase Encoding

- Gradient (e.g. in $y$ direction) is applied for a fixed time $T_y$, before readout
- Results in phase angle $\phi_p = \gamma G_{pe} y T_y$
Start with gradient that turns the transversal magnetization by 360°

- Double the gradient with each measurement, until neighboring pointers face in opposite direction
- Same procedure for negative gradients
- Results in a frequency comb, scanning through all needed spatial frequencies
- Unlike frequency encoding, phase encoding can be done in multiple dimensions.
- As many measurements as scanned rows needed.
- Basically frequency encoding in pseudo time.
**k-Space**

- k-Space is spanned by $k_x = \gamma G_{fe} t$ and $k_y = \gamma G_{pe} T_y$
- Every measurement is positioned in k-space

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**Measured signal in k-space**

$$S(k_x, k_y) = \int \int dx dy \, M'_T(x, y) \exp(-ik_x x - ik_y y)$$

- Measurement in k-space is the 2D Fourier transform of transverse magnetization!
To reconstruct image, k-space needs to be adequately sampled.

Every traversal to fill k-space is equal (e.g. Cartesian, spiral)
Cartesian Sampling Example
Fourier Transformation

- Pixel in k-space is a frequency information
- Pixels close to the center represent low spatial frequencies, far away from the center high spatial frequencies
- Selective sampling of k-space can be used to reduce measuring time

\[
\Delta z = \frac{\Delta \omega_{hf}}{\gamma G_{ss}} \quad \Delta y = \frac{\pi}{\gamma G_{pe}^\text{max} T_y} \quad \Delta x = \frac{\pi}{\gamma G_{fe} T_x}
\]
Fourier Transformation
Fourier Transformation
Low-Pass Filtering
High-Pass Filtering
Noise
3D Scanning

- Many individual measurement on different slices
  - Mainly used for thick slices
  - Faster, but worse signal-to-noise ratio compared to three dimension k-space scanning
- Three dimensional k-space scanning
  - Add third gradient using phase encoding

\[ S(k_x, k_y, k_z) = \iiint dx dy dz \ M'_T(x, y, z) \exp(-i(k_xx + k_yy + k_zz)) \]
Thank you for your attention!


Sources of Figures

- Slide 2: Adaptation from [3], page 56.
- Slide 3: From [3], page 13.
- Slide 5: Adaptation from [3], page 115.
- Slide 6: Adaptation from [3], page 114.
- Slide 7: Adaptation from [3], page 122, adaption from [3], page 109.
- Slide 8: Adaptation from [3], page 124.
- Slide 9: Adaptation from [3], page 126.
- Slide 11: Adaptation from [3], page 124.
Sources of Figures

- Slide 12: From [3], page 121.
- Slide 14: Adaptation from [3], page 124.
- Slide 16: From [3], page 129.