Magnetic Resonance Imaging
From Signal to Image

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Outline

1. Spatial encoding
   - Basic idea: gradient fields
   - Slice selection
   - Frequency encoding
   - Phase encoding

2. Using the k-space
   - The signal as a Fourier transform
   - Accessing points & sampling k-space
   - Restrictions

3. Contrast
   - Taking advantage of relaxation
   - Crucial parameters
Basic idea

- Starting point: Oscillatory signal with frequency $\omega_0 = \gamma \cdot B_0$

- Spins flipped into x-y-plane, realign with $B_0$ ($T_1$) and defocus ($T_2$)

- Received signal = superposition of signals emitted in different locations

- => no spatial information provided

Make precession frequency a function of space!
Gradient Fields

- Gradient = magnetic field that varies with space
- Here: linear change
- $B = B_0 + \hat{G} \cdot \vec{r}$ where $\hat{G} = \nabla B = \text{const.}$
- Only magnitude changes, not direction

Source: Tony Stöcker, Lecture on MRI [1]
Slice Selection

- Apply z-gradient during RF-pulse
- Only certain bandwidth $\Delta \omega$ fulfills resonance condition, since $\omega(z) = \gamma B_0 + \gamma G_z z$
- Slice thickness given by

$$\Delta z = \frac{2\pi \Delta f}{\gamma G_z}$$

Source: O. Dössel: Bildgebende Verfahren in der Medizin [2]
Slice Selection

- Ideal case: rectangular frequency response

- Only valid for small tip angles

Source: Tony Stöcker, Lecture on MRI [1]
Slice Selection

- Problem: spins defocus along $\Delta z$
- Apply inverse gradient to refocus

Source: O. Dössel: Bildgebende Verfahren in der Medizin [2]
Frequency encoding

• x-gradient during readout

• \[ \omega(x) = \omega_0 + \gamma G_x x \]

• Signal decomposed into spectral components

Source: O. Dössel: Bildgebende Verfahren in der Medizin [2]
Phase encoding

- y-gradient before readout
- during readout: same $\omega$ but different phase
- $\Delta \phi(y, \tau_y) = \gamma G_y y \tau_y$
- Repeat for different $G_y$

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The signal as a Fourier transform

-\( s(t) = \int d^3r \, M(\vec{r}, t) = \int d^3r \, M_0(\vec{r}) \, e^{-i\omega_0 t} \)
-\( = \iint dx\,dy \, m(x, y) \, e^{-i\omega_0 t} \)
-\( s(G_y, \tau_y, t) = \iint dx\,dy \, m(x, y) \, e^{-i\omega_0 t} \, e^{-i\gamma G_y y \tau_y} \)

- No relaxation, no gradients
- Since
\[
\int_{z_0 - \frac{\Delta z}{2}}^{z_0 + \frac{\Delta z}{2}} M_0(\vec{r}) dz = m(x, y)
\]
- Apply \( y \)-gradient for time \( \tau_y \) before readout:
\[
\omega_0 \rightarrow \omega_0 + \gamma G_y y
\]
The signal as a Fourier transform

- Apply x-gradient for time $\tau_x$ during readout:
  \[ \omega(y) \rightarrow \omega(y) + \gamma G_x x \]

- \[ s(G_y, \tau_y, G_x, \tau_x, t) = \int \int dx dy \ m(x, y) \ e^{-i\omega_0 t} e^{-i\gamma G_y y \tau_y} e^{-i\gamma G_x x \tau_x} \]

- Problem: oscillating term $e^{-i\omega_0 t}$
  => demodulation using quadrature detector
The signal as a Fourier transform

Source: O. Dössel: Bildgebende Verfahren in der Medizin [2]
The signal as a Fourier transform

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The signal as a Fourier transform

- Demodulating signal equivalent to changing to rotating frame
- \( \rightarrow \text{get rid of } e^{-i\omega_0 t} \)

Source: Tony Stöcker, Lecture on MRI [1]
The signal as a Fourier transform

\[ s(G_x, \tau_x, G_y, \tau_y) = \int \int dx dy \ m(x, y) e^{-i \gamma G_y y \tau_y} e^{-i \gamma G_x x \tau_x} \]

Now define \( k_i = \gamma G_i \tau_i \)

\[ \Rightarrow s(k_x, k_y) = \int \int dx dy \ m(x, y) e^{-i k_x x} e^{-i k_y y} \]
\[ = FT\{m(x, y)\} \]
Sampling k-space

Source: Tony Stöcker, Lecture on MRI [1]
Sampling k-space

Source: Tony Stöcker, Lecture on MRI [1]
Sampling k-space

- $k_i = \gamma G_i \tau_i$

- $k$ only depends on area under gradient curve!

- Theoretically arbitrary $\vec{k}$ accessible, if convenient $G_i, \tau_i$ are chosen
Sampling k-space

Source: Tony Stöcker, Lecture on MRI [1]
Sampling k-space

Spin echo sequence

Source: Tony Stöcker, Lecture on MRI [1]
Sampling k-space

Spin echo sequence

Source: Tony Stöcker, Lecture on MRI [1]
Sampling $k$-space

Radial imaging

Spiral imaging

Source: Tony Stöcker, Lecture on MRI [1]
Restrictions

- k-space actually unlimited continuum

- One can only sample
  1. a finite part and
  2. discrete points of k-space!
Restrictions: Finite part

\[ m(x, y) = \delta(x - x_0, y - y_0) \]

\[ \tilde{m}(x, y) \propto \text{sinc} \left( 2k_{x,\text{max}}(x - x_0) \right) \text{sinc} \left( 2k_{y,\text{max}}(y - y_0) \right) \]

\[ M(k_x, k_y) = \exp(-i(k_x x_0 + k_y y_0)) \]

\[ s(k_x, k_y) = M(k_x, k_y) \cdot \text{rect} \left( \frac{k_x}{2k_{x,\text{max}}}, \frac{k_y}{2k_{y,\text{max}}} \right) \]

Source: Tony Stöcker, Lecture on MRI [1]
Restrictions: Finite part

- Rayleigh criterion yields expression for resolution:
- \[ \Delta x_i = \frac{1}{2k_{i,\text{max}}} \]

Source: Tony Stöcker, Lecture on MRI [1]
Restrictions: Discrete Sampling

- Only discrete measurements
- Points separated by $\Delta k_x, \Delta k_y$
- Nyquist: $f_{\text{sample}} > 2f_{\text{max}}$

$\Rightarrow$ Field of View limited!

$$FOV_i = \frac{1}{\Delta k_i}$$

Source: Tony Stöcker, Lecture on MRI [1]
Restrictions: Discrete Sampling

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Taking advantage of relaxation

- Relaxation neglected so far → contrast provided only by proton density

- Taking relaxation into account provides additional sources for contrast
Crucial parameters

- Spins realign with $B_0$ over time ($T_1$) => multiple 90°-pulses necessary
- Time between two pulses: $TR$
- Spins defocus due to $\omega(r^*)$ => refocus by gradient, 180° pulse, etc
- Time from 90° pulse to „echo“: $TE$

Source: Tony Stöcker, Lecture on MRI [1]
Crucial parameters

- \( s(t) \propto M_0 \exp\left(-\frac{TE}{T_2}\right) \exp\left(1 - \frac{TR}{T_1}\right) \) for spin echo

- \( TE \ll T_2, TR \approx T_1 \): „\( T_1 \)-weighted“

- \( TE \approx T_2, TR \gg T_1 \): „\( T_2 \)-weighted“

- \( TE \ll T_2, TR \gg T_1 \): „PD-weighted“
Taking advantage of relaxation

$T_1$ decay

Source: Tony Stöcker, Lecture on MRI [1]
Taking advantage of relaxation

$T_2$ decay

Source: Tony Stöcker, Lecture on MRI [1]
Crucial Parameters

Source: Tony Stöcker, Lecture on MRI [1]
Summary

- Spatial encoding achieved by gradient fields (slice selection, frequency encoding, phase encoding)
- Demodulated signal $= \text{Fourier transform of magnetisation}$
- $k$-space sampling can be achieved by choosing different gradient strengths $G$ and application times $\tau$
- Finite and discrete sampling lead to limited resolution and limited FOV
- Different contrasts achievable by choosing convenient $TR, TE$
Sources

[1] Tony Stöcker: Lecture on MRI
Thank you for your attention!